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ELECTROMAGNETIC FIELD STRENGTH ABOVE AN ATMOSPHERIC SURFACE DUCT

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ABSTRACT: *The paper presents a method which allows the calculation of the atmospheric distortion of radar pules, provided that the influence of the atmosphere is to transfer the transmitted signal through a duct. The polarization of the primary sources, whose moment varies arbitrarily in time, is chosen in such a way that it allows the exact determination of the electric field strength at some field point above the duct layer. We can determinate the transient behaviour of the electric field strength at any distance above the duct.*

KEYWORDS: Electromagnetic field, Atmospheric surface duct

INTRODUCTION

Historically, in the problem of electromagnetic radiation from a vertical dipole situated at a certain height h above a plane earth, all field quantities are usually assumed to vary harmonically in time. Sommerfeld[1], calculated the electromagnetic radiation from an electric vertical dipole, located above the plane interface of two media. Many writers, Wait[2],Moore[3] and Durrani[4] have considered this problem, the aim of the present work is to extend the study-state to transient excitation when no restrictions on the distance between receiving and transmitting ends are made. Two integral transforms are applied to analyze the transient field of vertical electric dipole above a dielectric layer. The distinction of different cases where the distance between the receiving and transmitting ends are great and lesser than the total reflection distance studies by Abo-Seliem[5]. The problem has been studied by Arutaki and chiba[6] and Abo-Seliem[7]. This integral is estimated by using the steepest descent method, along the coutour Γ and around the branch-cuts, from the obtained result. The Saddle point method show that the reflected waves and integrals Abo-Seliem[8], the component of the electric field strength is also arbitrary for the excitation function $\mathbf{F(t) = t}$ at some fixed but arbitrary position from the point of observation in the half-space.

FORMULATION OF THE PROBLEM

As show in Fig.1, the duct model of Kahan and Eckart[9]. A dielectric layer is assumed of relative permittivity ε_1 over laying an infinitely conducting plane earth which is confined by the plane $\mathbf{z = 0}$ of a rectangular coordinate system. The source of the field is assumed to be a vertical electric dipole in the medium 1 at the point $\mathbf{x = y = 0}$, $\mathbf{z = d > 0}$ whose moment is given by $\tilde{\pi}_e = (0,0,F(t)\delta(x,y,z-d))$, t being the time variable and δ the three dimensional-distribution. Regarding $\mathbf{F(t)}$, we make the assumptions $\mathbf{F(t) = 0}$ for $\mathbf{t \le 0}$ and $\frac{dF(t)}{dt} = 0$ for $\mathbf{t = 0}$.

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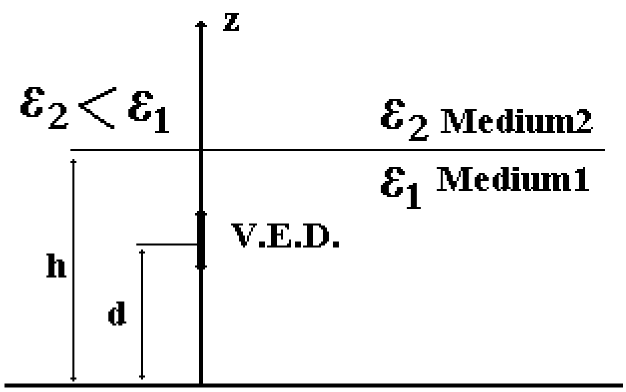
Fig.1: Geometric of the problem



METHOD OF SOLUTION

The starting point is the wave equation for the electrical field $\vec{\mathbf{E}} = (\mathbf{x}, \mathbf{y}, \mathbf{z}; \mathbf{t})$ in the two media:

$$[\nabla^2 - v_i^{-2}\partial_t^2]\vec{E} = \begin{cases} 0 \quad \text{for } i = 2 \\ \mu_0 D_t^2 F(t)\delta(x,y,z-d)\vec{e}_z - \dfrac{F(t)}{\varepsilon_0 \varepsilon_i}\nabla\partial_z\delta(x,y,z-d) \quad \text{for } i = 1 \end{cases} \qquad (1)$$

where $\mathbf{v}_i$ denotes the phase velocity of medium $\mathbf{i}$, $\vec{\mathbf{e}}_z$ is a unit vector in the z-direction. The application of a Laplace transform in time and two-dimensional. Fourier transform horizontal coordinates x,y leads under consideration of the initial boundary and transform of $\vec{\mathbf{E}} = (\mathbf{x}, \mathbf{y}, \mathbf{z}; \mathbf{t})$ being the variable in the transform space, we get for $\mathbf{h} < \mathbf{z} < \infty$

$$[\frac{\partial^2}{\partial z^2} - \gamma_i^2 s^2]F^{(i)}(\alpha, \beta, z, s) = \begin{cases} 0 \quad \text{for } i = 2 \\ f(s)[\dfrac{-js(\alpha\vec{e}_x + \beta\vec{e}_y)}{\varepsilon_0\varepsilon_i}\dfrac{\partial}{\partial z}\delta(z-d) + \\ (\Gamma_0 s^2\delta(z-d) - \dfrac{\partial^2}{\partial z^2}\dfrac{\delta(z-d)}{\varepsilon_0\varepsilon_u})\vec{e}_z] \quad \text{for } i = 1 \end{cases} \qquad (2)$$

where $\mathbf{j}^2 = -1$, $\gamma^2 = (\alpha^2 + \beta^2 + v_i^{-2})$, $\mathbf{i} = 1,2$ with $\Re e(\gamma_i) \geq 0$ in the medium 1. This an integral representation result of the Laplace transform of electric field in terms of two-dimensionals inverse Fourier integral.

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$$E_z^{(i)}(x,y,z;s)=\frac{s^3 f(s)}{8\pi^2}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\alpha^2+\beta^2)[\frac{\exp(-s\gamma_i|z-d|)}{\gamma_i}+\frac{(1+c_{12}\exp(-2s\gamma_i(h-d)))\exp(-s\gamma_i(z+d))}{\gamma_i(1+c_{12}\exp(-2s\gamma_i h))}+ c_{12}\frac{(1+c_{12}\exp(-2s\gamma_i d))\exp(-s\gamma_i(2h-z-d))}{\gamma_i(1+c_{12}\exp(-2s\gamma_i h))}\}\exp(js(\alpha x+\beta y))d\alpha d\beta \quad (3)$$

with the reflection coefficient at the upper duct boundary: $c_{12}=\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}$ here α and β are variables in the transform space of the two-dimensional Fourier transform $f(s)$ is the Laplace transform $F(t)$.

DISCUSSION OF THE INTEGRALS

To discuss the function $E_z^{(i)}(x,y,z;s)$ which is stated in (3) from mathematical and physical points of view. If follows that by using polar coordinates:

$x=\rho\cos\varphi$, $y=\rho\sin\varphi$ and $\alpha=\lambda\cos\varphi'$, $\beta=\lambda\sin\varphi'$

where $\lambda^2=(\alpha^2+\beta^2)$, $\rho^2=(x^2+y^2)$ and $d\alpha d\beta=\lambda d\lambda d\varphi'$

The evaluation of the double integral (3) is a difficult task. Therefore, using Bessel integral representation:

$$E_z^{(i)}(x,y,z;s)=\frac{s^3 f(s)}{8\pi^2}\int_0^{\infty}[\frac{\exp(-s\gamma_i|z-d|)}{\gamma_i}+\frac{(1+c_{12}\exp(-2s\gamma_i(h-d)))\exp(-s\gamma_i(z+d))}{\gamma_i(1-c_{12}\exp(-2s\gamma_i h))}+ c_{12}\frac{(1+c_{12}\exp(-2s\gamma_i d))\exp(-s\gamma_i(2h-z-d))}{\gamma_i(1-c_{12}\exp(-2s\gamma_i h))}\lambda^3 J_1(\lambda sp)d\lambda \quad (4)$$

where $J_1(\lambda sp)$ is the Bessel function of order one.

The second term in (4) can be solved in [8], the third term will be dealt with as it represents the secondary field. Therefore:

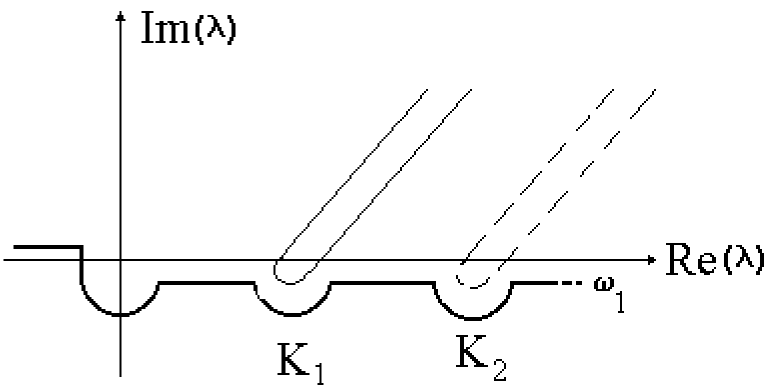
$$E_z^{(i)}(\rho,\theta,z;s)=\frac{-sf(s)}{\pi}\int_0^{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}\lambda^3 J_0(\lambda sp)d\lambda \quad (5)$$

where $F_1(\lambda s)=(\gamma_1-\gamma_2)\exp(-s\gamma_1(z-d))+(\gamma_1-\gamma_2)\exp(-s\gamma_1(z+2h-d))$ (6)

$$M_1(\lambda s)=\gamma[(\gamma_1+\gamma_2)-(\gamma_1-\gamma_2)\exp(-2s\gamma_1 h)] \quad (7)$$

To discuss the integral (5) we have to investigate the singularities in the denominator of the integral. The integral has four values that correspond to the four combinations of signs of γ_1 and Riemann surface also has four sheets. To insure the convergence of our integrals, we require that the path of integration, at infinity,

Fig.2: Branch cuts, the steepest descent paths and poles in the appositive □-plane.



should be on the permissible sheet only as previously demonstrated by Kahan and Eckart[9], the poles, the branch cuts an the branch point which are suitable for operating the integration will be also determinate. We find two branch at $\lambda = k_1$ and $\lambda = k_2$ where $v_i^{-1} = jk$, an infinity number of poles on the upper Riemann sheets Fig.(2). illustrates these two branch cuts and the steepest descent paths.

Next, $J_1(\lambda sp)$ is written in terms of Hankel function $H_1^{(1)}(\lambda sp)$ to change the semi-infinite integral (5) into a fully infinite integral.

$$E_z^{(i)}(\rho,\theta,z;s) = \frac{-sf(s)}{\pi}\int_{-\infty}^{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}\lambda^2 H_1^{(1)}(\lambda sp)d\lambda \qquad (8)$$

in (8) can be evaluated along the contour ω from $-\infty$ to ∞, and its values goes around the poles and the branch cut Eq.(8) then takes the form:

$$E_z^{(i)}(\rho,\theta,z;s) = -2jsf(s)[\sum\frac{F_1(\lambda s)}{M_1(\lambda s)}H_1^{(1)}(\lambda sp)\lambda^2;\lambda_k] + [-\frac{sf(s)}{2\pi}\int_{\infty}\frac{F_1(\lambda s)}{M_1(\lambda s)}H_1^{(1)}(\lambda sp)\lambda^2 d\lambda] \qquad (9)$$

where $n'(\alpha)$ are the eigenvalues of the poles of the integral and λ_k is the solution of the poles equation.

$$M_1(\lambda s) = \gamma_1(\lambda_k s)[(\gamma_1(\lambda_k s) + \gamma_2(\lambda_k s)) - (\gamma_1(\lambda_k s) - \gamma_2(\lambda_k s))\exp(-2s\gamma_1(\lambda_k s)h)] = 0 \qquad (10)$$

in the first term of (9), substituting the value $\lambda = \lambda_k$.

$$D^k(\lambda_k s) = -2jsf(s)[\frac{H_1^{(1)}(\lambda sp)\lambda^2 F_1(\lambda_k s)}{M_1(\lambda_k s)};\lambda_k] \qquad (11)$$

where $M_1(\lambda_k s) = [\frac{\partial M(\lambda_k s)}{\partial \lambda}]_{\lambda=\lambda_k}$

Next, we can estimate the second term in (9) by using Saddle-point method. The Hankel function can be transformed into the asymptote expansions as is well known[8]

$$\mathbf{H}_n^{(1)}(\mathbf{p}) = \sqrt{\frac{2}{\pi\rho}}\exp(\mathbf{j}(\mathbf{p}-\pi(2\mathbf{n}-1)/4)) \tag{12}$$

from (6) and (7), we can get the following

$$\mathbf{I} = \int\sqrt{\frac{2\lambda}{\pi\mathbf{p}\gamma_1}}\exp(\mathbf{j}(\mathbf{sp}-\pi/4))\,\mathbf{A}(\lambda\mathbf{s})\exp(-\mathbf{s}\gamma_1(\mathbf{z}+\mathbf{d}))\mathbf{d}\lambda \tag{13}$$

where

$$\mathbf{A}(\lambda\mathbf{s}) = \frac{\lambda^2[(\gamma_1-\gamma_2)\exp(-\mathbf{s}\gamma_1(\mathbf{z}-\mathbf{d}))+(\gamma_1-\gamma_2)]\exp(-\mathbf{s}\gamma_1(\mathbf{z}+2\mathbf{h}-\mathbf{d})}{\gamma_1[(\gamma_1+\gamma_2)-(\gamma_1-\gamma_2)\exp(-2\mathbf{s}\gamma_1\mathbf{h})]} \tag{14}$$

we let $\rho = \mathbf{R}\sin\alpha$, $(\mathbf{z}+\mathbf{d}) = -\mathbf{R}\cos\alpha$ and $\gamma_i^2 + \mathbf{k}_i^2 = \lambda^2$,

$$\mathbf{I} = \int\sqrt{\frac{2}{\pi\mathbf{sR}\sin\alpha}}\exp(\mathbf{j}(\mathbf{Rsg}(\lambda\mathbf{s})-\pi/4))\,\Phi(\lambda\mathbf{s})\mathbf{d}\lambda \tag{15}$$

where $\mathbf{g}(\lambda\mathbf{s}) = \lambda\sin\alpha - \mathbf{j}\sqrt{\lambda^2-\mathbf{k}_i^2}\cos\alpha$ and $\Phi(\lambda\mathbf{s}) = \mathbf{A}(\lambda\mathbf{s})\sqrt{\frac{\lambda}{\gamma_1^2}}$

Thus, the Saddle-point $\lambda = \lambda_s$ for the integral is determinated by[9].

$$\mathbf{g}'(\lambda\mathbf{s}) = \sin\alpha - \mathbf{j}\frac{\lambda\cos\alpha}{\sqrt{\gamma_1^2}} \tag{16}$$

Therefore $\lambda = \mathbf{k}_1\sin\alpha$ **(17)**

Then, the integral (15) is evaluated as follows:

$$\mathbf{I} = \mathbf{A}(\lambda_k\mathbf{s})\frac{2\exp(\mathbf{isk}_i\mathbf{R})}{\mathbf{sR}} \tag{18}$$

If the height duct is h=20m and the difference of relative permutations in the boundary $\Delta\varepsilon = 4.10^{-4}$,the height of the primary source and the point of observation are taken as z=d=15m ,the $n_1 = \frac{V_1}{V_2} = 1.00033$,if r=4000m.

Figures describes the relationship between Im/y/ and t at $t < 6 \times 10^{-3}$ *s .The absolute value of the z-component of the electric field strength increasing with increase the time . The absolute value of the z-component of the electric field strength is increasing when the spherical distance between the source and the point of observation is very small.*($|E|\ \alpha\ \frac{1}{R}$). *We note that The absolute value of the of the electric field strength is dependent of* R *. At* $t >$

6×10^{-3} s .The absolute value of the electric field strength is constant for each values of R = 5 km ,10 km, 15 km , 20 km and 25 km .

We note that :- the saturation curve when the time increase.

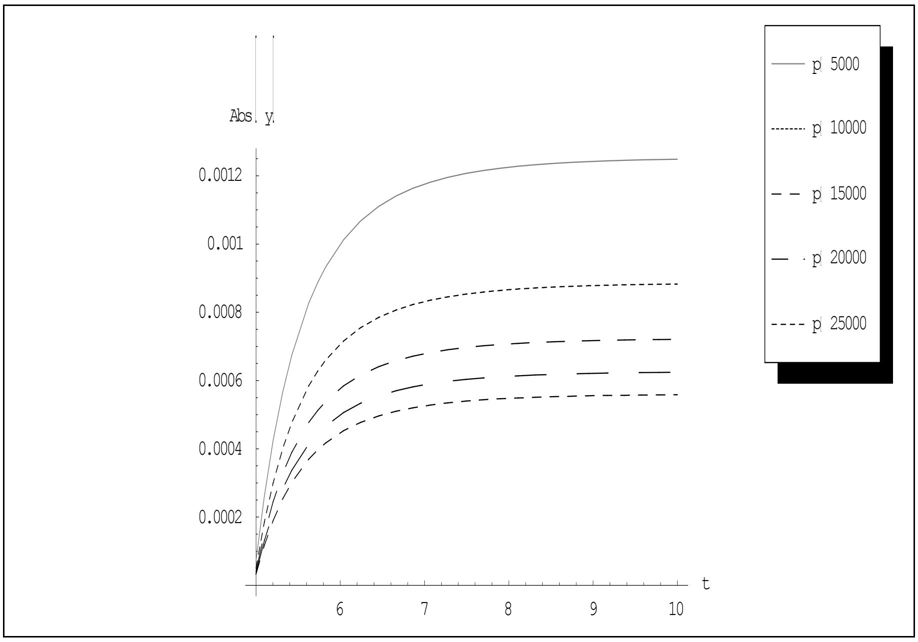
Figure (3): describes the relationship between Log |Abs|y|| and t.

1- At time < 5×10^{-3} s .The value of Log of the electric field strength is negative (-ve) value .

2- At time(t) < 6×10^{-3} s . Log value of the electric field strength increasing by increase the time .

3- At time(t) > 6×10^{-3} s . Log value of the electric field strength is constant (+ve) for all value of .

Fig.(3) describes the relationship between Abs[y]and t



CONCLUSION

A theoretical study for computing the electromagnetic field from a Hertizan vector in the ionosphere is presented. The solution is valid for arbitrary distances between receiving and transmitting ends for a source position. The Saddle point method is used to compute the problem.

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