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MHD FORCED CONVECTION IN A HORIZONTAL DOUBLE – PASSAGE WITH UNIFORM WALL HEAT FLUX

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ABSTRACT: *The MHD forced convection in a horizontal double – passage with uniform wall heat flux has been studied by taking into account the effect of magnetic parameter* M*. The flow of the fluid is assumed to be laminar, two – dimensional, steady and fully developed. The fluid is incompressible and the physical properties are constants. The walls are kept at uniform heat fluxes. A uniform magnetic field* M *is applied and is assumed undisturbed as the induced magnetic field is neglected by assuming small Reynolds number* Re*.The solutions of the velocity profiles* u_1 *and* u_2 *and the temperature profiles* θ_1 *and* θ_2 *are obtained analytically. It is pointed out that the effect of MHD forced convection in a horizontal double – passage enhances the effect of flow at the passage 2 than passage 1.*

KEYWORDS: Magneto hydrodynamic (MHD), Convection, Reynolds number, Heat flux.

INTRODUCTION

The Study of Magneto hydro dynamics forced convection in a horizontal double – passage with uniform wall heat flux has found in applications in several different systems such as the cooling of nuclear reactors, cooling of electronic devices, the solar energy collection, temperature plasmas etc.A comprehensive review of the study of MHD flows in relations to the applications to the above areas has been made by several authors. Tamad and Samad (2010) studied and analyzed the radiation and viscous dissipation effects on a steady two – dimensional magneto hydrodynamics free convection flow along a stretching sheet with heat generation. The non – linear ordinary differential equations governing the flow field under consideration have been transformed by a similarity transformation into a system of non – linear ordinary differential equations and then solved numerically by applying Nachtsheim – Swigert shooting iteration technique together with six order Runge – Kutta integration Scheme. Resulting non – dimensional velocity, temperature and concentration profiles are the presented graphically for different values of the parameters of physical engineering interest.

Alim et al (2007) investigated the pressure work and viscous dissipation effects on MHD natural convection along a Sphere. The laminar natural convection flow from a sphere immersed in a viscous incompressible fluid in the presence of magnetic field has been considered in this investigation.

The convective heat transfer may be enhanced in a horizontal channel by using rough surfaces, inserts, swirl flow device, turbulent promoter, etc. (1995). Unfortunately, most of these methods cause a considerable drop in the pressure. Guo et al. (1998) have suggested that the convective heat transfer could be enhanced by using special inserts. These inserts are designed to increase the included angle between the velocity vector and the temperature gradient vector, rather than to promote turbulence. So, the heat transfer is considerably enhanced with as little pressure drop as possible.

Cheng et al. (1989) have studied the effect of plane baffle, which is used as an insert, on the fully developed laminar convection in a horizontal channel. The uthors have determined, in closed forms, the Nusselt number and temperature profiles for the channel under asymmetric heating. They have concluded that the presence of the baffle may lead to an enhancement of heat transfer between the walls and fluid, according to the baffle position.

Cheng et al. have neglected the heat transfer due to the energy generated by viscous dissipation. Although viscous dissipation is usually neglected in low – speed and low – viscosity flows through conventionally sized channels of short length-to-width ratio is large (1998). For double-passage channels, the length-width ratio becomes large as the baffle becomes near the wall. So viscous dissipation may become important.

Gau et al (1999) studied secondary flow and enhancement flow and enhancement in horizontal parallel – plate and convergent channels heating from below. Jin et al (1996) experimentally studied the unstable mixed convection of air in a bottom heated horizontal rectangular duct. Upstream migration of heat during combined convection in a horizontal parallel plate duct was investigated by Ingham et al (1996).

Nyce et al (1992) studied mixed convection in a horizontal rectangular channel – experimental and numerical velocity distributions. Transient analysis on the onset of thermal instability in the thermal entrance region of a horizontal parallel plate channel was studied by Lee and Hwang (1991)

For the fully developed laminar duct flow, Guo et al (1998) observed that Nu for the case of isoflux thermal boundary condition is greater than Nu for the case of isothermal boundary condition. This can be explained based on the concept of included angle between the velocity and temperature gradients vectors. This angle is larger at isoflux thermal boundary condition than at isothermal boundary condition. Therefore, they stated that changing the thermal boundary condition could enhance the convective heat transfer.

Incropera et al (1998) studied the effects of wall heat flux distribution on laminar mixed convection in the entry region of a horizontal rectangular duct. Development of laminar mixed convection in a horizontal channel with uniform bottom heating was presented by Mahaney et al (1987). Also, Osborne and Incropera (1985) studied laminar mixed convection heat transfer for flow between horizontal parallel plates with asymmetric heating.

Saleh and Hashim (2009) analysed flow reversal phenomena of the fully – developed laminar combined free and forced MHD convection in a vertical plate – channel where the effect of viscous dissipation is taken into account.

Ingham et al (1995) studied the recirculating laminar mixed convection in a horizontal parallel plate duct. Kennedy and Zebib (1983) studied the combined free and forced convection between horizontal parallel plates. Vorticity – velocity method for Graetz problem with the effect of natural convection in a horizontal rectangular channel with uniform wall heat flux was studied by Chou and Hwang (1987)

Ranuka et al (2009) studied the MHD effects of unsteady heat convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plates temperatures oscillates with the same frequency as that of variable suction velocity with soret effects. The governing equations are solved numerically by using implicit finite difference method.

Abou- Ellail and Morcos (1983) studied the buoyancy effects in the entrance region of horizontal rectangular channels. Buoyancy effects on laminar heat transfer in the entrance region of horizontal rectangular channels with uniform heat flux for large prandtl number fluids was investigated by Cheng et al (1972)

Huang and Lin (1983) studied buoyancy induced flow transition in mixed flow of air through a bottom heated horizontal rectangular duct. Numerical solution for combined free and forced laminar convection in horizontal rectangular channels is investigated by Cheng et al (1969). Incropera and Schutt presented a numerical simulation of laminar mixed convection in the entrance region of horizontal rectangular ducts.

Also, El-Din (2002) investigated the effect of viscous dissipation on fully developed laminar mixed convection in a horizontal double-passage analytically. The channel is divided into two passages by means of a thin, perfectly conductive plane baffle and the walls have different uniform heat fluxes. Velocity and temperature profiles and the Nusselt number on the hot wall have been determined in closed forms. Results show that the Brinkman number has a significant effect on the dimensionless temperature, specially when the baffle is near to any channel's walls. The variations of the Nusselt number on the hot wall with Brinkman number depends on the baffle position.

Therefore, the present work is devoted to study, analytically, MHD forced convection in a horizontal double – passage with uniform wall heat flux.

Problem Formulation

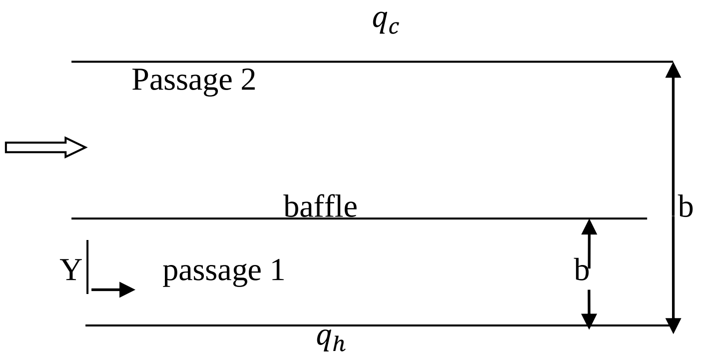
The geometry of the problem is shown in figure 1. The flow of the fluid is assumed to be laminar, two – dimensional, steady and fully developed. The fluid is incompressible and the physical properties are constants. The walls are kept at uniform heat fluxes. A uniform magnetic field M is applied and is assumed undisturbed as the induced magnetic fiel is neglected by assuming small Reynolds number Re.

With the above assumption, the momentum equation is given by

$$\frac{d^2U_i}{dY^2} - M^2[U_i + Re] = \frac{1}{\mu}\frac{dP_i}{dX} \tag{1}$$

Where $i = 1$ refers to stream 1 and $i = 2$ stream 2.

Figure 1. The schematic diagram of the horizontal double – passage channel



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The relevant boundary conditions are

$$y = 0, \qquad u_1 = 0$$
$$y = b^*, \qquad u_1 = u_2 = 0$$
$$y = b, \qquad u_2 = 0 \qquad (2)$$

Introducing the following dimensionless quantities.

$$X = \frac{x}{bR_e}, \quad Y = \frac{y}{b}, \quad U = \frac{u}{u_0}, \quad P = \frac{p}{\rho u_0^2}, Re = \frac{u_0 b}{v}$$

Where the reference velocity u_0 is defined as

$$u_2 = \frac{1}{b}\int_0^b u dy \qquad (3)$$

The boundary Momentum equation becomes

$$\frac{d^2U_i}{dY^2} - M^2[U_i + R] = \frac{1}{\mu}\frac{dP_i}{dX} \qquad (4)$$

The pressure gradient in equation (4) is assume to be constants, i.e.

$$\frac{dP_i}{dX} = \gamma_i \qquad (5)$$

The dimensionless boundary conditions are

$$Y = 0, \qquad U_1 = 0$$
$$Y = Y^*, \qquad U_1 = U_2 = 0$$
$$Y = 1, \qquad U_2 = 0 \qquad (6)$$

Solution of the Problem

Solving equation (4) for $i = 1$ subject to the boundary condition s in equation (6), we have

$$U_1 = \left(\frac{\gamma_1}{M^2} + R\right)\left\{\frac{\left(1-e^{-MY^*}\right)e^{MY}}{\left(e^{MY^*}-e^{-MY^*}\right)} - \frac{\left(1-e^{MY^*}\right)e^{-MY^*}}{\left(e^{MY^*}-e^{-MY^*}\right)} - 1\right\} \tag{7}$$

Similarly, for $i = 2$, the solution of equation subject to boundary conditions in equation (6) is

$$U_2 = \left(\frac{\gamma_2}{M^2} + Re\right)\left\{\frac{\left(e^{-MY^*}-e^{-M}\right)e^{MY}}{\left(e^{M-MY^*}-e^{MY^*-M}\right)} + \frac{\left(e^{M}-e^{MY^*}\right)e^{-MY}}{\left(e^{M-MY^*}-e^{MY^*-M}\right)}\right\} \tag{8}$$

Conservation of mass considered at any section of the channel passages gives

$$\int_0^{Y^*} U_1 dY = Y^* \tag{9}$$

And

$$\int_0^1 U_2 dY = 1 - Y^* \tag{10}$$

Substituting equation (7) into equation (9), we obtain

$$\gamma_1 = M^2\left\{\frac{MY^*\left(e^{MY^*}-e^{-MY^*}\right)-Re\left\{2\left(e^{MY^*}-e^{-MY^*}\right)-4-MY^*\left(e^{MY^*}-e^{-MY^*}\right)\right\}}{2\left(e^{MY^*}+e^{-MY^*}\right)-4-MY^*\left(e^{MY^*}-e^{-MY^*}\right)}\right\} \tag{11}$$

$$\gamma_2 = M^2\left\{\frac{2M(1+Re)(1-Y^*)\left(e^{M-MY^*}-e^{MY^*-M}\right)-Re\left[\left(e^{M-MY^*}+e^{MY^*-M}\right)-4\right]}{2\left(e^{M-MY^*}+e^{MY^*-M}\right)-4+M(Y^*-1)\left(e^{M-MY^*}-e^{MY^*-M}\right)}\right\} \tag{12}$$

Substituting equation (11) into equation (7), we have

$$U_1 = \frac{\left[MY^{*\left(e^{MY^*}-e^{-MY^*}\right)-Re\left\{\left(e^{MY^*}-e^{-MY^*}\right)-2\right\}}\right]\left\{\left(1-e^{-MY^*}\right)e^{MY}-\left(1-e^{MY^*}\right)e^{-MY}-\left(e^{MY^*}-e^{-MY^*}\right)\right\}}{\left[\left(e^{MY^*}+e^{-MY^*}\right)-2-MY^*\left(e^{MY^*}-e^{-MY^*}\right)\right]\left(e^{MY^*}-e^{-MY^*}\right)} \tag{13}$$

Substituting equation (12) into equation (8), we obtain the following velocity profile

$$U_2 = \frac{\left[-2\left\{M(Y^*-1)\left(e^{M-MY^*}-e^{MY^*-M}\right)+\left(e^{M-MY^*}+e^{MY^*-M}-2\right)\right\}+Re\left\{M(Y^*-1)\left(e^{M-MY^*}-e^{MY^*-M}\right)-4\left(1-e^{M-MY^*}\right)\right\}\right]\left\{\left(e^{-MY^*}-e^{-M}\right)e^{MY}+\left(e^{M}-e^{-MY^*}\right)e^{-MY}-\left(e^{M-MY^*}-e^{MY^*-M}\right)\right\}}{\left[M(Y^*-1)\left(e^{M-MY^*}-e^{MY^*-M}\right)-4\left(1-e^{M-MY^*}\right)\right]\left[e^{M-MY^*}-e^{M-MY^*}\right]} \tag{14}$$

The energy equation of the fully developed flow, taking into account the effect of viscous dissipation, is

$$k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{du}{dy}\right)^2=\rho c_\rho u\frac{\partial T}{\partial x} \tag{15}$$

With boundary conditions

$$y=0,\quad \frac{dT}{dy}=\frac{-q_h}{k}$$

$$y=b^*,\quad T_1=T_2$$

$$y=b,\quad \frac{\partial T}{\partial y}=\frac{q_c}{k} \tag{16}$$

Integration of equation (15) with respect to y in the interval $0\to b$, using equation (3) yields

$$k\frac{\partial T}{\partial y}|_{y=b}-k\frac{\partial T}{\partial y}|_{y=0}+\mu\int_0^b\left(\frac{du}{dy}\right)^2=\rho c_\rho u_0 b\frac{\partial T}{\partial X} \tag{17}$$

Using the boundary conditions in equation (16), equation (17) takes the following form

$$\frac{\partial T}{\partial x}=\frac{1}{\rho c_p u_0 b}\left(q_c+q_h+\mu\int_0^b\left(\frac{\partial u}{\partial x}\right)^2 dy\right) \tag{18}$$

Substitution from equation (18) into equation (15) gives the energy equation, which can be written in the following dimensionless form,

Where,

$$\theta=\frac{T-T_0}{{q_h b}/{k}},\quad Br=\frac{\mu u_0^2}{q_h b},\quad R=\frac{q_c}{q_h},\quad \frac{\partial\theta}{\partial y}=\frac{\frac{\partial T}{\partial Y}}{{q_h b}/{k}},\quad U=\frac{u}{u_0},\quad \frac{\partial^2\theta}{\partial Y^2}=\frac{\frac{\partial^2 T}{\partial y^2}}{{q_h b}/{k}},\quad \frac{\partial U}{\partial Y}=\frac{\frac{du}{dy}}{u_0}$$

Therefore, the required result is

$$\frac{\partial^2\theta}{\partial Y^2}+Br\left(\frac{\partial U}{\partial Y}\right)^2=\left[1+Re+Br\int_0^1\left(\frac{\partial U}{\partial Y}\right)^2 dY\right]U+BrM^2\int_0^1[U+Re]^2\,dY \tag{19}$$

Where,

$$Br=\frac{\mu u_0^2}{q_h b},\qquad 1=\frac{1}{b^2},\quad Re=\frac{q_c}{b^2 q_h}$$

The reference temperature T_0 is defined as

$$T_0=\frac{1}{b}\int_0^1 T\,dY \tag{20}$$

The dimensionless boundary conditions are

$$Y=0,\quad \frac{\partial\theta}{\partial Y}=-1$$

$$Y=Y^*,\quad \theta_1=\theta_2$$

$$Y=1,\quad \frac{\partial\theta}{\partial Y}=Re \tag{21}$$

Differentiating and squaring of equations. (13) and (14) with respect to Y, the terms $\int_0^1 \left(\frac{\partial u}{\partial Y}\right)^2 dY$ and $\int_0^1 [U + Re]^2 dY$ in equation (19) can be obtained and hence, the integration in the right – hand side can be evaluated.

$$\int_0^1 \left(\frac{\partial u}{\partial Y}\right)^2 dY + \int_0^1 [U + Re]^2 dY = D_2^2 M\left\{\left(1 + e^{-MY^*}\right)\left(e^{2MY^*} - 1\right) + \left(1 - e^{MY^*}\right)\left(1 - e^{-2MY^*}\right) + 4M\left(e^{-MY^*} - e^{MY^*}\right)Y^*\right\} + D_2^2 \frac{M}{2}\left\{\left(e^{MY^*} - e^{-M}\right)\left(e^{2M} - e^{2MY^*}\right) - \left(e^{M} - e^{MY^*}\right)\left(e^{-2MY^*} - e^{-2M}\right) - 4M\left(e^{M-MY^*} + e^{MY^*-M}\right)(Y^* - 1)\right\} + \frac{D_3^2}{2M}\left(e^{2MY^*} - 1\right) + \frac{D_4^2}{2M}\left(1 - e^{-2MY^*}\right) + \frac{D_6}{M}\left(e^{MY^*} - 1\right) + \frac{D_7}{M}\left(1 - e^{-MY^*}\right) + \frac{D_9^2}{2M}\left(e^{2M} - e^{2MY^*}\right) + \frac{D_{10}^2}{2M}\left(e^{-2MY^*} - e^{-2M}\right) + \frac{D_{12}}{M}\left(e^{M} - e^{MY^*}\right) + \frac{D_{13}}{M}\left(e^{-MY^*} - e^{-M}\right) + (D_8 - D_{14})Y^* \qquad (22)$$

Where, $D_1 = \frac{\{M(MY^* - Re(1 - MY^*)) + Re(1 - M^4 Y^*)\}}{(1 - M^4 Y^*)(e^{MY^*} + e^{-MY^*})}$

$$D_2 = \frac{\left[-4M(Y^*-1)\left(e^{M-MY^*} - e^{MY^*-M}\right) + \left(e^{M-MY^*} + e^{MY^*-M} - 2\right) + Re\left\{eM(Y^*-1)\left(e^{M-MY^*} - e^{MY^*-M}\right) - 4\left(1 - e^{MY^*-M}\right)\right\}\right]}{\left[M(Y^*-1)\left(e^{M-MY^*} - e^{MY^*-M}\right) - 4\left(1 - e^{M-MY^*}\right)\right]\left(e^{M-MY^*} - e^{MY^*-M}\right)}$$

$$D_3 = D_1\left(1 + e^{-MY^*}\right)$$

$$D_4 = D_1\left(1 - e^{-MY^*}\right)$$

$$D_5 = Re - D_1\left(e^{MY^*} + e^{-MY^*}\right)$$

$$D_6 = 2D_3 D_5$$

$$D_7 = 2D_4 D_5$$

$$D_8 = D_5^2 - 2D_3 D_4$$

$$D_9 = D_2\left(e^{-MY^*} + e^{-M}\right)$$

$$D_{10} = D_2\left(e^{M} - e^{MY^*}\right)$$

$$D_{11} = Re - D_2\left(e^{M-MY^*} + e^{MY^*-M}\right)$$

$$D_{12} = 2D_9 D_{11}$$

$$D_{13} = 2D_{10} D_{11}$$

$$D_{14} = D_{11}^2 - 2D_9 D_{10}$$

Equation (19) can be rewritten as

$$\frac{\partial^2\theta}{\partial Y^2} + Br\left(\frac{\partial U}{\partial Y}\right)^2 + BrM^2[U+Re]^2 = UZ \tag{23}$$

Where,

$$Z = 1 + S + D_2^2 MBr\{(1+e^{-MY^*})(e^{2MY^*}-1) + (1-e^{MY^*})(1-e^{-2MY^*}) + 4M(e^{-MY^*} - e^{MY^*})Y^*\} + BrD_2^2\frac{M^3}{2}\{(e^{MY^*} - e^{-M})(e^{2M} - e^{2MY^*}) - (e^{M} - e^{MY^*})(e^{-2MY^*} - e^{-2M}) - 4M(e^{M-MY^*} + e^{MY^*-M})(Y^*-1)\} + \frac{D_3^2}{2M}(e^{2MY^*}-1) + \frac{D_4^2}{2M}(1-e^{-2MY^*}) + \frac{D_6}{M}(e^{MY^*}-1) + \frac{D_7}{M}(1-e^{-MY^*}) + \frac{D_9^2}{2M}(e^{2M}-e^{2MY^*}) + \frac{D_{10}^2}{2M}(e^{-2MY^*}-e^{-2M}) + \frac{D_{12}}{M}(e^{M}-e^{MY^*}) + \frac{D_{13}}{M}(e^{-MY^*}-e^{-M}) + (D_8 - D_{14})Y^* \tag{24}$$

Integrating equation (23) twice with respect to Y for the two passages of the channel, using equation (21), gives

$$\theta_1 = D_1 Z\left\{\frac{(1+e^{-MY^*})e^{MY}}{M^2} - \frac{(1-e^{MY^*})}{M^2} - \frac{(e^{MY^*}-e^{-M})Y^2}{2}\right\} + F_1 Y + F_2 - BrD_1^2M^2\left\{\frac{(1+e^{-MY^*})e^{MY}}{4M^2} + (e^{-MY^*}-e^{MY^*})Y^2 + \frac{(1-e^{MY^*})e^{-2MY}}{4M^2}\right\} - BrM^2\left\{\frac{D_3^2e^{MY}}{4M^2} + \frac{D_4^2e^{-2MY}}{4M^2} + \frac{D_6^2e^{MY}}{M^2} - \frac{D_7}{M^2} + \frac{D_8Y^2}{2}\right\} \tag{25}$$

$$\theta_2 = Z\left(\frac{D_9e^{MY}}{M^2} + \frac{D_{10}e^{-MY^*}}{M^2} + \frac{D_{11}Y^2}{2}\right) + GY + G_1 - BrD_2^2M^2\left\{\frac{(e^{-MY^*}-e^{-M})e^{2MY}}{4M^2} - (e^{M-MY^*} - e^{MY^*-M} - 2)Y^2 + \frac{(e^{M}-e^{MY^*})e^{-2MY}}{4M^2}\right\} - BrM^2\left\{\frac{D_9^2e^{2MY}}{4M^2} + \frac{D_{10}^2e^{-2MY}}{4M^2} + \frac{D_{12}e^{MY}}{M^2} + \frac{D_{13}e^{-MY}}{M^2} + \frac{D_{14}Y^2}{2}\right\} \tag{26}$$

The constant G is given by

$$G = R - Z\left\{\frac{D_9e^{M}}{M} - \frac{D_{10}e^{-M}}{M} + D_{11}\right\} + BrD_2^2M^2\left\{\frac{(e^{-MY^*}-e^{-M})}{2M} - 2(e^{M-MY^*} - e^{MY^*-M} - 2) - \frac{(e^{M}-e^{MY^*})e^{-2M}}{2M}\right\} + BrM^2\left\{\frac{D_9^2e^{2M}}{2M} - \frac{D_{10}^2e^{-2M}}{2M} + \frac{D_{12}e^{M}}{M} - \frac{D_{13}e^{-M}}{M} + D_{14}\right\} \tag{27}$$

Substitution from equations (25) and (26) into equation (21b) gives the following relation.

$$G_1 = G_2 + F_1 \tag{28}$$

Where,

$$G_2 = D_1 Z\left\{\frac{\left(1+e^{-MY^*}\right)e^{MY^*}}{M^2} - \frac{\left(1-e^{MY^*}\right)e^{-MY^*}}{M^2} - \frac{\left(e^{MY^*}+e^{-M}\right)Y^{*2}}{2}\right\} + F_1 Y^* - BrD_1^2 M^2\left\{\frac{\left(1+e^{-MY^*}\right)e^{2MY^*}}{4M^2} + \left(e^{-MY^*} - e^{MY^*}\right)Y^{*2} + \frac{\left(1-e^{MY^*}\right)e^{-2MY^*}}{4M^2}\right\} - BrM^2\left\{\frac{D_3^2 e^{2MY^*}}{4M^2} + \frac{D_4^2 e^{-2MY^*}}{4M^2} + \frac{D_6^2 e^{MY^*}}{M^2} - \frac{D_7}{M^2} + \frac{D_8 Y^{*2}}{2}\right\} - Z\left(\frac{D_9 e^{MY^*}}{M^2} + \frac{D_{10}e^{-MY^*}}{M^2} + \frac{D_{11}Y^{*2}}{2}\right) - GY^* + BrD_2^2 M^2\left\{\frac{\left(e^{-MY^*}-e^{-M}\right)e^{2MY}}{4M^2} - \left(e^{M-MY^*} + e^{MY^*-M} - 2\right)Y^{*2} + \frac{\left(e^{M}-e^{MY^*}\right)e^{-2MY^*}}{4M^2}\right\} + BrM^2\left\{\frac{D_9^2 e^{2MY^*}}{4M^2} + \frac{D_{10}^2 e^{-2MY^*}}{4M^2} + \frac{D_{12}e^{MY^*}}{M^2} + \frac{D_{13}e^{-MY^*}}{M^2} + \frac{D_{14}Y^{*2}}{2}\right\} \quad (29)$$

The constant F_1 can be obtained by use of equation (20). Introducing the dimensionless parameters into equation (20) gives

$$\int_0^1 \theta dY = 0 \quad (30)$$

Thus,

$$\int_0^{Y^*} \theta_1 dY + \int_{Y^*}^1 \theta_2 dY = 0 \quad (31)$$

Integration of equation (25) with respect to Y in the interval $0 \to Y^*$gives

$$\int_0^{Y^*} \theta_1 dY = F_2 Y^* + F_3 \quad (32)$$

Where,

$$F_3 = D_1 Z\left\{\frac{\left(1+e^{-MY^*}\right)\left(e^{MY^*}-1\right)}{M^3} - \frac{\left(1-e^{MY^*}\right)\left(e^{-MY^*}-1\right)}{M^3} - \frac{\left(e^{MY^*}+e^{-M}\right)Y^{*3}}{6}\right\} + \frac{F_1 Y^{*2}}{2} - BrD_1^2 M^2\left\{\frac{\left(1+e^{-MY^*}\right)\left(e^{2MY^*}-1\right)}{8M^3} + \left(e^{-MY^*} - e^{MY^*}\right)Y^{*3} + \frac{\left(1-e^{MY^*}\right)\left(e^{-2MY^*}-1\right)}{8M^3}\right\} - BrM^2\left\{\frac{D_3^2\left(e^{2MY^*}-1\right)}{8M^3} + \frac{D_4^2\left(e^{-2MY^*}-1\right)}{8M^3} + \frac{D_6^2\left(e^{MY^*}-1\right)}{M^3} - \frac{D_7 Y^*}{M^2} + \frac{D_8 Y^{*4}}{8}\right\} \quad (33)$$

Integration of equation (26) with respect to Y in the interval $Y^* \to 1$ gives

$$\int_{Y^*}^1 \theta_2 dY = F_1(1 - Y^*) + G_4 \quad (34)$$

Where

$$G_4 = Z\left\{\frac{D_9\left(e^{M}-e^{MY^*}\right)}{M^3} - \frac{D_{10}\left(e^{-M}-e^{-MY^*}\right)}{M^3} + \frac{D_{11}\left(1-Y^{*3}\right)}{6}\right\} + \frac{G\left(1-Y^{*2}\right)}{2} -$$

$$BrD_2^2M^2\left\{\frac{\left(e^{-MY^*}-e^{-M}\right)\left(e^{2M}-e^{2MY^*}\right)}{8M^3}-\frac{\left(e^{M-MY^*}+e^{MY^*-M}-2\right)\left(1-Y^{*3}\right)}{3}-\frac{\left(e^{M}-e^{MY^*}\right)\left(e^{-2M}-e^{-2MY^*}\right)}{8M^3}\right\}-$$
$$BrM^2\left\{\frac{D_9^2\left(e^{2M}-e^{2MY^*}\right)}{8M^3}-\frac{D_{10}^2\left(e^{-2M}-e^{-2MY^*}\right)}{8M^3}+\frac{D_{12}\left(e^{M}-e^{MY^*}\right)}{M^2}-\frac{D_7\left(e^{-2M}-e^{-MY^*}\right)}{M^3}+\frac{D_{14}Y^{*3}}{6}\right\}$$

(35)

Adding equations (32) to equations (34) gives the constants F_1

$$F_1=-(F_2+G_4) \tag{36}$$

The Nusselt number for the hot wall is

$$Nu_h=\frac{1}{\theta_h-\theta_{b1}} \tag{37}$$

The dimensionless bulk temperature for passage 1 is defined as

$$\theta_{b1}=\int_0^{Y^*}U_1\theta_1 dY\bigg/\int_0^{Y^*}U_1 dY=\int_0^{Y^*}U_1\theta_1/Y^* \tag{38}$$

DISCUSSIONS OF RESULTS

To study the effect of MHD forced convection in a double – passage, the velocity and the temperatures profiles at the passages 1 and 2 are depicted graphically for different values of the magnetic parameter M with the help of MATLAB.

Figures 1 and 2 demonstrate the variations of the velocities U_1 *and* U_2 at the passages 1 and 2 respectively for different values of M with small Reynolds number $Re=1$ and small Brinkman number $Br=1$. Figures 3 and 4 also show the variations of temperatures θ_1 *and* θ_2 at the passages 1 and 2 respectively for different values of M.

It is observed from figure 1 that that the velocity U_1 decreases with decrease in the magnetic parameter while figure 2 shows that the velocity U_2 increases with increase in the magnetic parameter. In figure 3, it is seen that the temperature decreases uniformly with decrease in particular values of M i.e. when $M=0.12,0.14,0.16$ *and* 0.18. But there is deflection when $M=0.2, 0.22, 0.24, 0.26, 0.28$ *and* 0.3, that is due to increase in the magnetic parameter . Figure 4 shows that the temperature θ_2 increases with decrease in the magnetic parameter.

To this effect of the magnetic parameter on the velocity and temperatures profiles, it is pointed out that the effect of MHD forced convection in a horizontal double – passage enhances the effect of flow at the passage 2 than passage 1.

Figure 1: Effect of M on velocity profile U_1

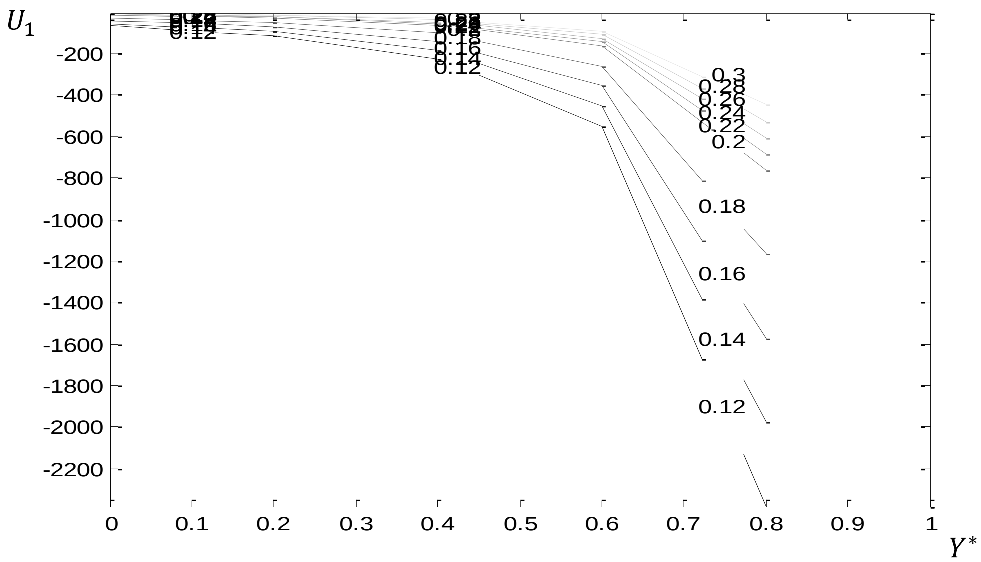


Figure 2 : Effect of M on the Velocity profile U_2

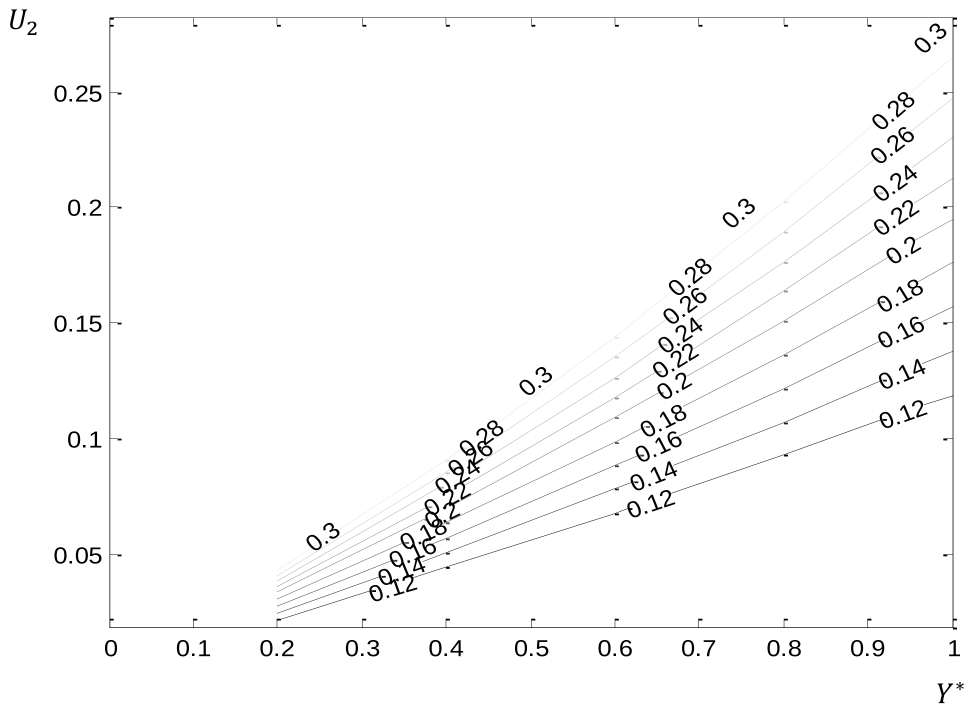


Figure3: Effect of M on the Temperature θ_1 profile.

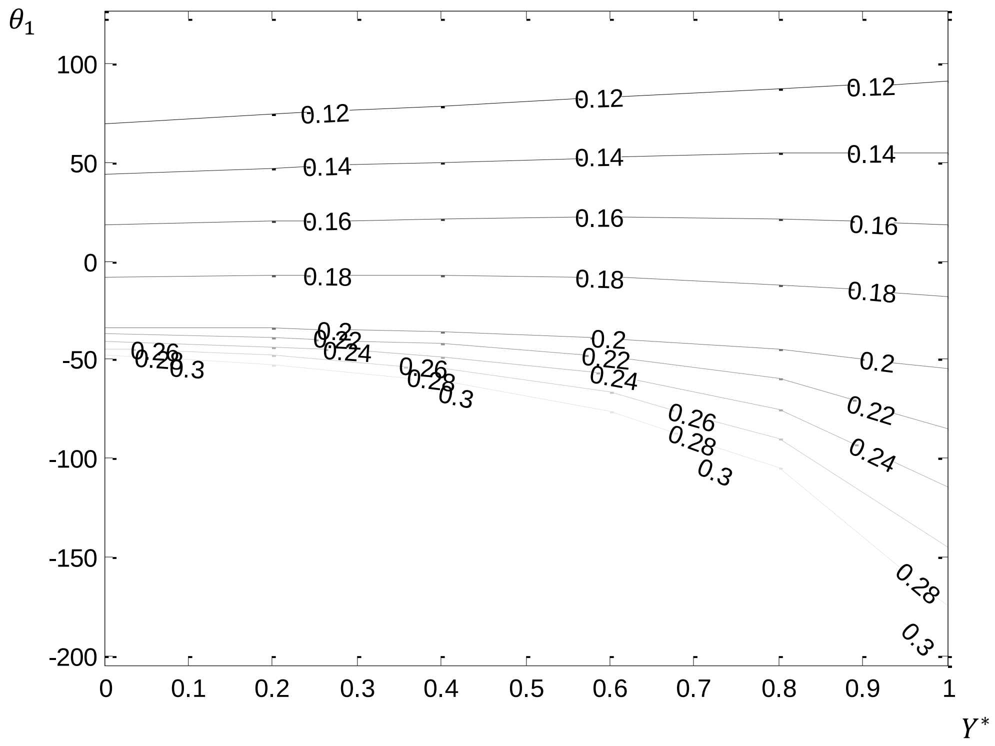
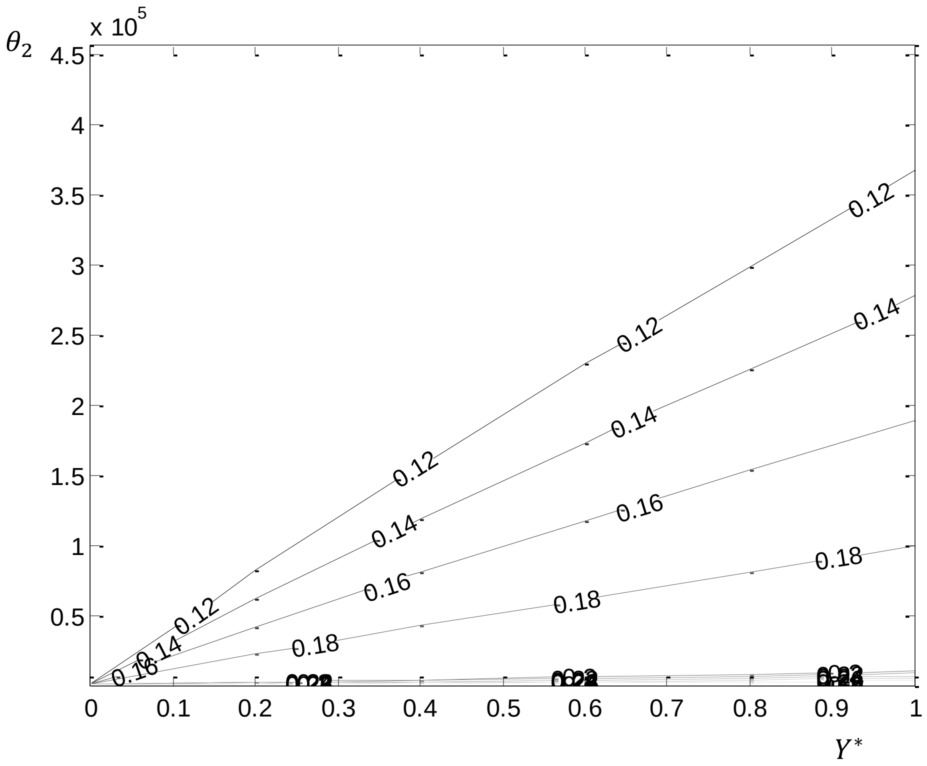


Figure 4. Effect of M on the Temperature profile θ_2.



SUMMARY AND CONCLUSION

The MHD forced convection in a horizontal double – passage with uniform wall heat flux has been studied by taking into account the effect of magnetic parameter M. The flow of the fluid is assumed to be laminar, two – dimensional, steady and fully developed. The fluid is incompressible and the physical properties are constants. The walls are kept at uniform heat fluxes. A uniform magnetic field M is applied and is assumed undisturbed as the induced magnetic field is neglected by assuming small Reynolds number Re.

The governing equations (momentum and energy equations) have been written in a dimensionless form. The solutions of the velocity profiles $u_1 and\, u_2$ and the temperature profiles $\theta_1 and\, \theta_2$ are obtained analytically. It is pointed out that the effect of MHD forced convection in a horizontal double – passage enhances the effect of flow at the passage 2 than passage 1.

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