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# IMPACT OF NEW WATER SOURCES ON THE OVERALL WATER NETWORK: AN OPTIMISATION APPROACH

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**ABSTRACT:** *A mathematical programme is formulated for a water network with new water sources included. Salinity and water hardness are considered in the model which is later solved using Max- Min Ant System (MMAS) to assess the impact of new water sources to the total cost of the existing network. It is efficient to include new water sources if the distance is short and if there is high penatly associated with failure to meet demand. MMAS can be used to find the best water network which include the new water sources at a minimum cost.*

**KEYWORDS**: Impact, New Water Sources, Optimisation, MMAS

## INTRODUCTION

Water source choice depends on water availability, cost of operation and development, quality of water and adequacy. It is important to evaluate all alternative water sources to ascertain the cost associated with each source. Economic, environmental, engineering and energy factors must also be considered when choosing the source of water. In semi- arid countries, the allocation of water is a particular challenge as there are high costs associated with construction of water distribution systems (WDSs).

In recent years, several researchers have focused on the development of mathematical techniques to minimise the costs associated with constructing water distribution infrastructure. Research has been carried out on the implementation of Evolutionary Algorithms (EA) in various fields. Algorithms that were used include generic (Dandy et al. 1996, Savic and Walters 1997, Lippai et al.1999, Wu et al. 2001), Ant colony optimisation (Maier et al. 2003) and simulated annealing (Cunhaand Sousa 1999). The Genetic Algorithm was also applied in the autocalibration of a chlorine transport model (Kumar and Munallavi 2003). Ant System (AS) and Max-Min Ant System (MMAS) were applied to three WDS case studies (Zecchin et al 2003). In this research a model to predict the impact of new water sources is developed and the Max-Min Ant System (MMAS) is used to find the minimum cost.

The Max-Min Ant System (MMAS) an adaptation of the general Ant System, which was developed from the behaviour of ants when searching for food (Colorni et al. 1996). Ants deposit an aromatic substance, called a pheromone, when finding food. After some time the pheromone trail disappears if no other ants use the same path, resulting in the more frequented paths retaining a higher intensity of pheromone. The Ant System associates pheromone trails with the solution of combinatorial problems.

Evolutionary Algorithms (EAs) converge to sub-optimal solutions prematurely and this is a problem, especially in those cases that have a greater emphasis on exploitation (Zecchin et al 2003). MMAS was developed by Stutzle and Hoos in 2000 to overcome this problem. MMAS provides dynamically evolving bounds on the pheromone trail intensities such that the pheromone intensity on all paths is always within a specified limit of the path with the greatest pheromone intensity. All paths will have a non-trivial selection probability. MMAS uses upper and lower bounds to ensure pheromone intensities lie within a given range, which means $\tau_{min}(t) \leq \tau_{ij}(t) \leq \tau_{max}(t)$. The upper and lower limits are given respectively;

$$\tau_{max}(t) = \frac{1}{(1-\rho)}\left(\frac{1}{f(S^{gb}(t-1))}\right) \quad (1)$$

$$\tau_{\min(t)} = \frac{\tau_{\max(t)(1-\sqrt[n]{P_{best}})}}{(NO_{avg}-1)\sqrt[n]{P_{best}}} \quad (2)$$

where $\tau_{ij}(t)$ is the concentration of pheromone associated with edge $(ij)$ in iteration $t$, ρ is the coefficient representing pheromone persistence so $1-\rho$ represents the evaporation of trail and $0 \leq \rho \leq 1$. $P_{best}$ is the probability that the current global-best path, $S^{gb}(t)$, will be selected given that all non-global best edges have a pheromone level of $\tau_{min}(t)$ and all global-best edges have a pheromone level of $\tau_{max}(t)$, $n$ is the number of decision points and $NO_{avg}$ is the average number of edges at each decision point and $f(.)$ is the objective function (Zecchin et al 2003).

The best ant that is allowed to add pheromone may be the best iteration. The problem of stagnation is decreased by giving each connection a chance of being chosen. MMAS uses re-initialization of pheronome trails in order to increase evaporation of the solution. MMAS was used to find a general solution method for the multi-level capacitated lot-sizing and scheduling problem (Almeder 2010), in multi-objective problems (Pinto and Baran 2004) and routing problems (Sodsoon 2010).

## NEW WATER SOURCES AND PIPELINES

We can expand the existing water system to supplement demand but the economies of using the existing source should be evaluated using alternatives.

### Water salinity consideration

It is important to consider salinity levels from these sources in order to determine the actual costs of supplying the water from these sources. The variables are defined in Table 1 below.

**Table 1: Definitions of raw and treated water quantities, salinity levels, price and cost of water sources**

| Source $(n)$ | Water Quantity $(Q)$ | | Salinity Level $(S)$ | | Cost $(P)$ | |
|---|---|---|---|---|---|---|
| | Untreated | Treated | Initial | Current | Water source | Average Cost of desalination |
| Local source $(L)$ | $Q_L$ | $Q_{L_t}$ | $S_L$ | $S_{L_t}$ | $P_L$ | $P_{L_t}$ |
| Foreign $(F)$ | $Q_F$ | $Q_{F_t}$ | $S_F$ | $S_{F_t}$ | $P_F$ | $P_{F_t}$ |
| Aquifer $(G)$ | $Q_G$ | $Q_{G_t}$ | $S_G$ | $S_{G_t}$ | $P_G$ | $P_{G_t}$ |
| Rain Harvesting $(P)$ | $Q_P$ | $Q_{P_t}$ | $S_P$ | $S_{P_t}$ | $P_P$ | $P_{P_t}$ |
| $n$ sources of $j$ type | $Q_{n,j}$ | $Q_{n,j}^{tr}$ | $S_{n,j}$ | $S_{n,j}^{tr}$ | $P(S)_{n,j}$ | $P(S)_{n,j}^{tr}$ |

$$Q = S_L Q_L + S_F Q_F + S_G Q_G + S_P Q_P \tag{3}$$

We need desalination if the threshold has been surpassed. Thus

$$(Q_L - Q_{L_t})S_L + Q_{L_t} S_{L_t} \leq Q_L S_L \tag{4}$$

$$(Q_F - Q_{F_t})S_F + Q_{F_t} S_{F_t} \leq Q_F S_F \tag{5}$$

$$(Q_G - Q_{G_t})S_G + Q_{G_t} S_{G_t} \leq Q_G S_G \tag{6}$$

$$(Q_P - Q_{P_t})S_P + Q_{P_t} S_{P_t} \leq Q_P S_P \tag{7}$$

Using equation (4) to (7) if we have $n$ sources of $j$ type we get

$$(Q_{n,j} - Q_{n,j}^{tr})S_{n,j} + Q_{n,j}^{tr} S_{n,j}^{tr} \leq Q_{n,j} S_{n,j} \tag{8}$$

Let $S^*$ be the required salinity level of water suitable for consumption then,

$$S_L + S_F + S_G + S_P = S^*$$

If we are getting water from our local source only then

$$S_L = S^* \quad (9)$$

Using the same concept we used to get equation (9) it follows that

$$S_L = S_F = S_G = S_P = S^*$$

The amount of water supplied for consumption can be calculated as the sum of the differences between quantity of water from each source and the treated water multiplied by their salinity level. Therefore;

$$(Q_L - Q_{L_t})S_L + (Q_F - Q_{F_t})S_F + (Q_G - Q_{G_t})S_G + (Q_P - Q_{P_t})S_P + Q_{L_t}S_{L_t} + Q_{F_t}S_{F_t}$$

$$+Q_{G_t}S_{G_t} + Q_{P_t}S_{P_t} = [Q_L + Q_F + Q_G + Q_P]S^*$$

$$\Rightarrow Q_{Lt}S_{Lt} - Q_{Lt}S_L + Q_{Ft}S_{Ft} - Q_{Ft}S_F + Q_{Gt}S_{Gt} - Q_{Gt}S_G + Q_{Pt}S_{Pt} - Q_{Pt}S_P$$

$$+Q_LS_L + Q_FS_F + Q_GS_G + Q_PS_P = Q_LS^* + Q_FS^* + Q_GS^* + Q_PS^*$$

$$\Rightarrow Q_{Lt}(S_{Lt} - S_L) + Q_{Ft}(S_{Ft} - S_F) + Q_{Gt}(S_{Gt} - S_G) + Q_{Pt}(S_{Pt} - SP) + Q_LS_L + Q_FS_F$$

$$+Q_GS_G + Q_PS_P = Q_LS^* + Q_FS^* + Q_GS^* + Q_PS^* \quad (10)$$

From equation (10) it follows that

$$Q_L(S_L - S^*) + Q_F(S_F - S^*) + Q_G(S_G - S^*) + Q_P(S_P - S^*) = Q_{L_t}(S_L - S_{L_t})$$

$$+Q_{F_t}(S_F - S_{F_t}) + Q_{G_t}(S_G - S_{G_t}) + Q_{P_t}(S_P - S_{P_t}) \quad (11)$$

Hence we can determine the actual amount of water from each source that has been desalinated by making quantities from other sources to be equal to zero. Thus

$$Q_{L_t} = \frac{Q_L(S_L - S^*)}{S_L - S_{L_t}} \quad (12)$$

$$Q_{F_t} = \frac{Q_F(S_F - S^*)}{S_F - S_{F_t}} \quad (13)$$

$$Q_{G_t} = \frac{Q_G(S_G - S^*)}{S_G - S_{G_t}} \quad (14)$$

$$Q_{P_t} = \frac{Q_P(S_P - S^*)}{S_P - S_{P_t}} \quad (15)$$

The cost associated with desalination is important to make a decision on whether to use the new sources. Let $C_S$ be the total cost of water suitable for consumption after desalination and $C_{S_{tr}}$ be the total cost of desalination, then

$$C_{S_{tr}} = P_{Lt}(S_L - S_{L_t}) + P_{Ft}(S_F - S_{F_t}) + P_{Gt}(S_G - S_{G_t}) + P_{Pt}(S_P - S_{P_t}) \quad (16)$$

and

$$C_S = P_L(Q_L - Q_{L_t}) + Q_{L_t}P_{L_t} + P_F(Q_F - Q_{F_t}) + Q_{F_t}P_{F_t} + P_G(Q_G - Q_{G_t}) + Q_{G_t}P_{G_t}$$

$$+P_P(Q_P - Q_{P_t}) + Q_{P_t}P_{P_t} \quad (17)$$

If there are $n$ sources of $j$ type we can generalise (16) and (17) as

$$C_{S_{tr}} = \sum_n^N \sum_j^K P(S)_{n,j}^{tr}\left(S_{n,j} - S_{n,j}^{tr}\right) \quad (18)$$

$$C_S = \sum_{n=1}^N \sum_{j=1}^K \left(P(S)_{n,j}(Q_{n,j} - Q_{n,j}^{tr}) + Q_{n,j}^{tr}P(S)_{n,j}^{tr}\right) \quad (19)$$

**Water hardness consideration**

Water hardness is important to consider when considering an alternative source of water. Hardness is defined as the amount of minerals found in water and is usually reported as an equivalent quantity of calcium carbonate ($CaCO_3$). Hardness can be reduced by softening. The easiest way to test for water hardness is lather or frost test. If water is hard the soap will not lather easily. Hard water causes limescales in kettles and washing machines, but it does not have any health related problems if consumed. For urban use water hardness is a matter of concern due to the kind of utensils and equipments used that can lather.

The WHO guidelines in 2004 identified that water with a hardness of value 200 mg/l or higher will produce scale and soft water with a value of 100 mg/l or less will have a low buffering capacity and be more corrosive to pipes (WHO 2009). The level of $CaCO_3$ (mg/ l) regarded as hard to extremely hard differs from country to country. The average value is any value above 200mg/ l as $CaCO_3$ and this is the value that will be used in this research. We need to soften the water if the level is exceeded. We define quantity of water and hardness levels as shown in Table 2.

**Table 2: Definitions of raw and treated water quantities, hardness levels, price and cost of water sources**

| Source ($n$) | Water Quantity ($Q$) | | Hardness levels ($S$) | | Cost ($P$) | |
|---|---|---|---|---|---|---|
| | Untreated | Treated | Initial | Current | Water source | Average Cost of softening |
| Local source ($L$) | $Q_L$ | $Q_{L_t}$ | $H_L$ | $H_{L_t}$ | $P_L$ | $P_{L_t}$ |
| Foreign ($F$) | $Q_F$ | $Q_{F_t}$ | $H_F$ | $H_{F_t}$ | $P_F$ | $P_{F_t}$ |
| Aquifer ($G$) | $Q_G$ | $Q_{G_t}$ | $H_G$ | $H_{G_t}$ | $P_G$ | $P_{G_t}$ |
| Rain Harvesting ($P$) | $Q_P$ | $Q_{P_t}$ | $H_P$ | $H_{P_t}$ | $P_P$ | $P_{P_t}$ |
| $n$ sources of $j$ type | $Q_{n,j}$ | $Q_{n,j}^{tr}$ | $H_{n,j}$ | $H_{n,j}^{tr}$ | $P(H)_{n,j}$ | $P(H)_{n,j}^{tr}$ |

Softening the water is required if the water hardness passed the required level. Using the same concept as in equation (4) to (7) we have

$$(Q_L - Q_{L_t})H_L + Q_{L_t}H_{L_t} \leq Q_L H_L \quad (20)$$

$$(Q_F - Q_{F_t})H_F + Q_{F_t}H_{F_t} \leq Q_F H_F \tag{21}$$

$$(Q_G - Q_{G_t})H_G + Q_{G_t}H_{G_t} \leq Q_G H_G \tag{22}$$

$$(Q_P - Q_{P_t})H_P + Q_{P_t}H_{P_t} \leq Q_P H_P \tag{23}$$

If we have $n$ sources of $j$ type we get

$$(Q_{n,j} - Q_{n,j}^{tr})H_{n,j} + Q_{n,j}^{tr}H_{n,j}^{tr} \leq Q_{n,j}H_{n,j} \tag{24}$$

If we have water from $n$ sources of $j$ type and taking $200mg/l$ as the maximum level then

$$\sum_{n=1}^{N} \sum_{j=1}^{K} H_{n,j} \leq 200 \tag{25}$$

The cost associated with softening are

$$C_{H_{tr}} = P_{Lt}(H_L - H_{L_t}) + P_{Ft}(H_F - H_{F_t}) + P_{Gt}(H_G - H_{G_t}) + P_{Pt}(H_P - H_{P_t}) \tag{26}$$

$$C_H = P_L(Q_L - Q_{L_t}) + Q_{L_t}P_{L_t} + P_F(Q_F - Q_{F_t}) + Q_{F_t}P_{F_t} + P_G(Q_G - Q_{G_t}) + Q_{G_t}P_{G_t} + P_P(Q_P - Q_{P_t}) + Q_{P_t}P_{P_t} \tag{27}$$

We can generalise char and equation (27) as

$$C_{H_{tr}} = \sum_{n}^{N} \sum_{j}^{K} P(H)_{n,j}^{tr}\left(H_{n,j} - H_{n,j}^{tr}\right) \tag{28}$$

$$C_H = \sum_{n=1}^{N} \sum_{j=1}^{K} \left(P(H)_{n,j}(Q_{n,j} - Q_{n,j}^{tr}) + Q_{n,j}^{tr}P(H)_{n,j}^{tr}\right) \tag{29}$$

**Costs**

Define;

$n$ index denoting number of water sources

$j$ index denoting type of water source

$i$ index denoting reservoir

$t$ index denoting time period

$Q_{n,j}$ water quantity to be supplied (Decision variable)

$C(O)_{n,j}$ operational costs including cost of material, fuel and labour

$C(conv)_{n,j}$ conventional cost that is maintenance cost

$C(trans)_{n,j}$ transportation costs

$C(inst)_{n,j}$ cost of installing pumps, reservoirs and pipes

$Q_{n,t}$ water from $n$ sources during period $t$

$Q(cap)_{i,t}^{min}$ minimum storage capacity of reservoir $i$ at period $t$

$Q(cap)_{i,t}$ storage volume in reservoir $i$ at time $t$

$C_{i,t}$ cost of water storage at reservior $i$ at start of time period $t$

$d_{i,j}$ distance from source to reservoir

$Q(cap)_{i,t}^{max}$ maximum storage capacity of reservoir $i$ at period $t$

$C_{pi}$ investment on pipes cost

$C(pi)_{i,j}$ investment cost to reservoir $i$ from source of $j$ type

$C_p$ pumping cost

$C(p)_{n,j}$ the cost of pumping water from $n$ sources of type $j$

The pumping cost at each new supply node is represented by pumpcost and it is useful to evaluate the impact of new water sources. The cost of the new pipes is equal to sum of distance from new source to resevoir multiplied by pipe diameter of the respective source. The distance is calculated as

$$d_{i,j} = \sqrt{(x_{ij} - x_{ji})^2 + (y_{ij} - y_{ji})^2} \tag{30}$$

Equation (29) is the investment on pipes equation.

$$C_p = \sum_n^N \sum_j^J C(p)_{n,j} Q_{n,j} \tag{31}$$

$$C_{pi} = \sum_{i=1}^I \sum_{j=1}^J C(pi)_{i,j} d_{i,j} \tag{32}$$

Although new water sources can be identified, it is imperative to consider the existing water reservoirs capacity. We can get as much water as possible from new sources but with limited reservoir capacity. Our objective is to minimise cost of including the new water sources to the existing water network $Z$.

$$\begin{aligned} MinZ = & \sum_{n=1}^N \sum_{j=1}^K C(O)_{n,j} Q_{n,j} + \sum_{n=1}^N \sum_{j=1}^K \left(P(S)_{n,j}(Q_{n,j} - Q_{n,j}^{tr}) + Q_{n,j}^{tr} P(S)_{n,j}^{tr}\right) \\ & + \sum_{n=1}^N \sum_{j=1}^K \left(P(H)_{n,j}(Q_{n,j} - Q_{n,j}^{tr}) + Q_{n,j}^{tr} P(H)_{n,j}^{tr}\right) \\ & + \sum_{n=1}^N \sum_{j=1}^K C(conv)_{n,j} Q_{n,j} + d \sum_{n=1}^N \sum_{j=1}^K C(trans)_{n,j} Q_{n,j} \\ & + \sum_{n=1}^N \sum_{j=1}^K C(inst)_{n,j} Q_{n,j}^* - \sum_{i=1}^K \sum_{t=0}^T C_{i,t} Q(cap)_{i,t} \end{aligned} \tag{33}$$

$subjectto$:

$$(Q_{n,j} - Q_{n,j}^{tr}) S_{n,j} + Q_{n,j}^{tr} S_{n,j}^{tr} \leq Q_{n,j} S_{n,j} \tag{34}$$

$$(Q_{n,j} - Q_{n,j}^{tr}) H_{n,j} + Q_{n,j}^{tr} H_{n,j}^{tr} \leq Q_{n,j} H_{n,j} \tag{35}$$

$$\sum_{n=1}^{N} \sum_{j=1}^{K} H_{n,j} \leq 200 \tag{36}$$

$$C_p = \sum_{n}^{N} \sum_{j}^{J} C(p)_{n,j} Q_{n,j} \tag{37}$$

$$C_{pi} = \sum_{i=1}^{I} \sum_{j=1}^{J} C(pi)_{i,j} d_{i,j} \tag{39}$$

$$Q(cap)_{i,t}^{min} \leq Q(cap)_{i,t} \leq Q(cap)_{i,t}^{max} \tag{40}$$

$$Q_{n,j} \geq 0 \tag{41}$$

## MODEL IMPLEMENTATION

City of Bulawayo in Zimbabwe is facing water allocation challenges and it is imperative to focus on alternative water sources. Nyamandlovu aquifer and groundwater abstraction using boreholes are better alternatives to supplement the current water quantity. Desalination and softening were considered. Shortest distance between the new water sourcesand reservoir was used to compute the cost. Quantities to be abstracted from the aquifer was an estimate obtained from City of Bulawayo's water engineering department. Costs were also extracted from City of Bulawayo's master plan. Three boreholes, one in Entumbane, Mpopoma and city center respectively were considered in this research as source of ground water. (WHO 2011, 2009) salinity and hardness levels were considered as the levels the water to be supplied to consumers should exhibit. Table 3 shows the data that was used.

**Table 3: Water data (Source: City of Bulawayo Master plan)**

| Reservoir | Quantity Supplied [$m^3$/sec] (Distance[$m$]) | | | |
|---|---|---|---|---|
| | Nyamadhlovu Aquifer | Borehole 1 | Borehole 2 | Borehole 3 |
| Tulihill | 1.200 (53890.630) | | | |
| Criterion | 0.230 (52928.631) | | | |
| 6J | | | 0.420 (2051.828) | |
| Hillside | | | | 0.235 (854.400) |
| Riffle Range | 0.432 (55050.182) | | | |
| Magwegwe No. 8 | | 0.757 (3420.526) | -0.420 (2886.174) | |
| Woodvalle | | | | 0.257 (3008.322) |

MIDACO version 3.0 was used in MATLAB version 7.0.4 to solve the mathematical programme. We set $\alpha = 0.9$, $\beta = 0.7$ and $\rho = 0.7$ using sensitivity analysis of the parameters. Set $m = 80$, $Q = 2 \times 10^3$ and $P_{best} = 0.01$. Results shows that it costs

$1807.00 including cost of investment on pipes to meet demand at any given second. Accornding to the City of Bulawayo Engineering department' s master plan the penalty of failing to meet demand at given second is estimated to be $2300.00. Inclusion of new water sources prove to be necessary as the cost of connecting the new water sources to the existing water network cost less than the penalty. The best network design which consider distance between the nodes is presented in Table 4.

**Table 4: Summary of new source - reservoir water allocation**

| Node | Variable | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **Water cost** | **Pumping** | **Desalination** | **Softening** | **Capacity** | **Supply** | **Distance** | **Demand** |
| | ($\$/m^3$) | cost ($\$/m^3$) | cost ($\$/m^3$) | cost ($\$/m^3$) | ($m^3$) | ($m^3$) | $(x,y)$ | |
| Tuli Hill | | | | | 22000 | | (1200, 3600) | 25000 |
| Criterion | | | | | 31740 | | (4000,2200) | 40000 |
| 6J | | | | | 4500 | | (2000,2300) | 23000 |
| Hillside | | | | | 23720 | | (2300,3200) | 21100 |
| Rifle Range | | | | | 6750 | | (750, 2400) | 20000 |
| Magwegwe No. 8 | | | | | 4000 | | (700, 3000) | 45000 |
| Woodvalle | | | | | 2500 | | (900, 1200) | 5000 |
| aquifer | 0.49 | 1.2 | 0.20 | 0.10 | | 55000 | (40000,41000) | |
| Borehole 1 | 0.13 | 1.43 | 0.25 | 0.15 | | 15000 | (4000, 3900) | |
| Borehole 2 | 0.12 | 1.43 | 0.25 | 0.15 | | 16000 | (3500, 3700) | |
| Borehole 3 | 0.14 | 1.43 | 0.25 | 0.15 | | 17000 | (2000, 4000) | |

## CONCLUSION

MMAS is an effective optimisation algorithm to solve network problems. Impact of new water sources to the existing network's total cost can be found by implementing MMAS. The best network inclusive of the new sources can be found by using MMAS and it is efficient to include new water sources in a network if the distance between the new source and reservoir is short and if high penalty is associated with failure to meet demand.

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